Exercise C: Monte Carlo Integration

Benedikt Ehinger

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Video H Monte Carlo Integration

1 Monte Carlo Integration

1.1 Exercise 1

Work out how to generate random points, distributed uniformly in the square −1 < x < 1, −1 < y < 1. Generate 10,000 points and count how many are inside the circle of radius 1 and use this count to estimate π. How close is your estimate? Write a script to repeat the above 100 times and estimate the standard deviation of your estimates of π. Now repeat, but with 100,000 [1,000,000] per trial. How good does your accuracy get?

1.2 Exercise 2

We are now trying to integrate another function:
\[ \int_0^1 x^3 \, dx \]
or in R:

```r
f = function(x) { x^3 }
```

A simple analytic solution exists here: \( \int_{x=0}^{1} x^3 = \frac{1}{4} \) How good do you get? Plot the running mean over iterations as in the lecture video.

1.3 Exercise 3

We are now trying to integrate a more difficult function:
\[ f(x, y) = \frac{x^2 e^{-x^2} + x^4 e^{-y^2}}{x e^{-x^2}} \]

1Source: VNS Summer School Exercise Larry Maloney

2Hint: Think of the ratio of the areas \( \frac{A_1}{A_0} \)
\[ f = \text{function}(x,y)\{ y^2\exp(-y^2)+x^4\exp(-x^2)/(x\exp(-x^2)) \}\]
in the integral \( \int_{x=1}^{5} \int_{y=-5}^{5} \)

This is a 2D-Plot of the log of \( f(x,y) \). I.e. what we want to integrate over.

Run the estimation for 10,000 random samples and compare it to a grid-approximation (in matlab use \([x,y] = \text{meshgrid}(\text{dx},\text{dy});\text{functionhandle}(x,y)\), in r use \(\text{outer}(\text{dx},\text{dy},\text{functionhandle})\)).

Plot the running mean over iterations as in the lecture video.