Exercise C: Monte Carlo Integration

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Video H Monte Carlo Integration

1 Monte Carlo Integration

1.1 Exercise 1

[1]

Work out how to generate random points, distributed uniformly in the square $-1 <$ x < 1, $-1 <$ y < 1. Generate 10,000 points and count how many are inside the circle of radius 1 and use this count to estimate π. [2] How close is your estimate? Write a script to repeat the above 100 times and estimate the standard deviation of your estimates of π. Now repeat, but with 100.000 [1.000.000] per trial. How good does your accuracy get?

1.2 Exercise 2

We are now trying to integrate another function:

$\int_0^1 x^3$

or in R:

```
f = function(x){ x^3 }
```

A simple analytic solution exists here: $\int_{x=0}^1 x^3 = 1/4$ How good do you get? Plot the running mean over iterations as in the lecture video.

1.3 Exercise 3

We are now trying to integrate a more difficult function:

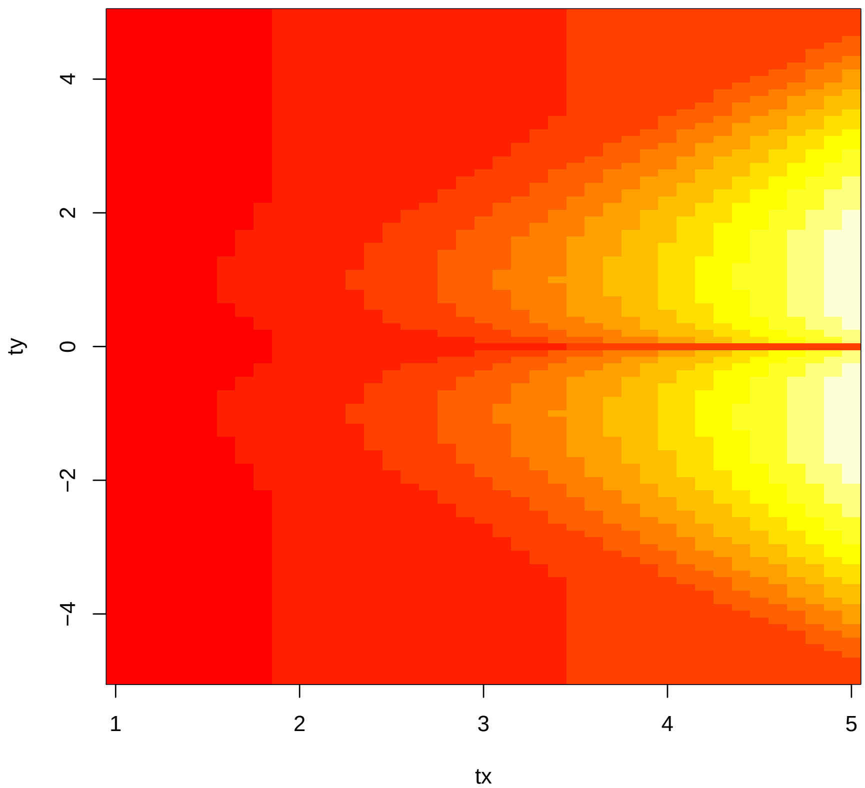
$f(x,y) = \frac{y^2 * e^{-y^2} + x^4 * e^{-x^2}}{x * e^{-x^2}}$

[1] Source: VNS Summer School Exercise Larry Maloney

[2] Hint: Think of the ratio of the areas $\frac{A_\circ}{A_\square}$

```r
f = function(x,y){ y^2*exp(-y^2)+x^4*exp(-x^2)/(x*exp(-x^2))}
```

in the integral $\int_{x=1}^{5}\int_{y=-5}^{5}$



This is a 2D-Plot of the log of f(x,y). I.e. what we want to integrate over.

Run the estimation for 10.000 random samples and compare it to a grid-approximation (in matlab use [x,y] = meshgrid(dx,dy);functionhandle(x,y), in r use outer(dx,dy,functionhandle)).

Plot the running mean over iterations as in the lecture video.