# Exercise A: Bayes Rule & Probability Distributions in R

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27. Oktober 2016

**Videos A+B:** Bayes Rule and Bayesian Statistics
**Optional (but recommended) Reading Material:**

- Statistical Rethinking Chapter 2
- Doing Bayesian Data Analysis Chapter 2+4

## 1 Probabilities

### 1.1 Exercise 1

[1] Which of the expressions below correspond to the statement: the probability of rain on Monday?

1. Pr(rain)
2. Pr(rain|Monday)
3. Pr(Monday|rain)
4. Pr(rain, Monday)/Pr(Monday)

Which of the following statements corresponds to the expression: Pr(Monday|rain)?

1. The probability of rain on Monday.
2. The probability of rain, given that it is Monday.
3. The probability that it is Monday, given that it is raining.
4. The probability that it is Monday and that it is raining.

Which of the expressions below correspond to the statement: the probability that it is Monday, given that it is raining?

1. Pr(Monday|rain)

[1] Source: Statistical Rethinking 2.6

2. Pr(rain|Monday)

3. Pr(rain|Monday) Pr(Monday)

4. Pr(rain|Monday) Pr(Monday)/Pr(rain)

5. Pr(Monday|rain) Pr(rain)/Pr(Monday)

## 1.2 Exercise 2

[2] Suppose we have a four-sided die from a board game. On a tetrahedral die, each face is an equilateral triangle. When you roll the die, it lands with one face down and the other three faces visible as a three-sided pyramid. The faces are numbered 1-4, with the value of the bottom face as x. The die might be manipulated and we consider the following three mathematical descriptions of the probabilities of observing $x$ (which could be 1 to 4)

- Model A: $p(x) = 1/4$.
- Model B: $p(x) = x/10$.
- Model C: $p(x) = 12/(25x)$.

For each model, determine the value of $p(x)$ for each value of $x$. Describe in words what kind of bias (or lack of bias) is expressed by each model.

## 1.3 Exercise 3

[3] Suppose we have the tetrahedral die introduced in the previous exercise, along with the three candidate models of the die's probabilities. Suppose that initially, we are not sure what to believe about the die. On the one hand, the die might be fair, with each face landing with the same probability. On the other hand, the die might be biased, with the faces that have more dots landing down more often. On yet another hand, the die might be biased such that more dots on a face make it less likely to land down. So, initially, our beliefs about the three models can be described as $p(A) = p(B) = p(C) = 1/3$ (Where capital letters denote models). Now we roll the die 100 times and find these results: #1's = 25, #2's = 25, #3's = 25, #4's = 25. Do these data change our beliefs about the models? Which model now seems most likely? Suppose when we rolled the die 100 times we found these results: #1's = 48, #2's = 24, #3's = 16, #4's = 12. Now which model seems most likely? Calculate the Bayes Factors for both observed data-sets. As a reminder the Bayes factor is defined as follows:

$logBF = log(\frac{P(data|A)}{P(data|B)}) = log(\prod_{x=1}^{4} \frac{P(data_x|A)}{P(data_x|B)})$

With

$P(data_x|A) = A(x)^{data_x}$

[2]Source: Doing Bayesian Data Analysis 2.4

[3]Source: Doing Bayesian Data Analysis 2.4

### 1.4 Exercise 4

Suppose there are two globes, one for Earth and one for Mars. The Earth globe is 70% covered in water. The Mars globe is 100% land. Further suppose that one of these globes—you don't know which—was tossed in the air and produced a "land" observation. Assume that each globe was equally likely to be tossed. Show that the posterior probability that the globe was the Earth, conditional on seeing "land" ($\Pr(\text{Earth}|\text{land})$), is 0.23.